

## SUPPRESSING LINEAR POWER ON DWARF GALAXY HALO SCALES

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## ABSTRACT

Recently it has been suggested that the dearth of small halos around the Milky Way arises due to a modification of the primordial power spectrum of fluctuations from inflation. Such modifications would be expected to alter the formation of structure from bottom-up to top-down on scales near where the short-scale power has been suppressed. Using cosmological simulations we study the effects of such a modification of the initial power spectrum. While the halo multiplicity function depends primarily on the linear theory power spectrum, most other probes of power are more sensitive to the non-linear power spectrum. Collapse of large-scale structures as they go non-linear regenerates a “tail” in the power spectrum, masking small-scale modifications to the primordial power spectrum except at very high- $z$ . Even the small-scale ( $k > 2 h \text{ Mpc}^{-1}$ ) clustering of the Ly- $\alpha$  forest is affected by this process, so that CDM models with sufficient power suppression to reduce the number of  $10^{10} M_{\odot}$  halos by a factor of  $\sim 5$  give similar Ly- $\alpha$  forest power spectrum results. We conclude that other observations that depend more directly on the number density of collapsed objects, such as the number of damped Ly- $\alpha$  systems, or the redshift of reionization may provide the most sensitive tests of these models.

*Subject headings:* cosmology:theory – large-scale structure of universe

## 1. INTRODUCTION

The field of physical cosmology has made rapid progress in the last decade, and a “standard model” is already beginning to emerge. Many of the main cosmological parameters are becoming known and there is good reason to believe that the measurements will be significantly improved, and the paradigm tested, in the next few years from observations of the Cosmic Microwave Background anisotropy and upcoming surveys of large-scale structure. While in broad outline the paradigm appears to work well, there are some discrepancies which indicate that revisions in our standard model may be required. In this paper we discuss several topics related to one of these issues: the lack of low mass halos (i.e. dwarf galaxies) in our local neighborhood (around the Milky Way).

This problem has been highlighted by several groups. Analytic arguments based on Press-Schechter (1974) theory were given by Kauffmann, White & Guiderdoni (1993), while Klypin et al. (1999) and Moore et al. (1999a) used very high resolution dark matter simulations. A summary of the situation has been given recently by Spergel & Steinhardt (1999) and Kamionkowski & Liddle (1999).

Within the Press-Schechter theory, and its extensions, the number density of halos of a given mass is related to the amplitude of the linear theory power spectrum on a scale proportional to  $M^{1/3}$ . For example  $10^{10} M_{\odot}$  halos in a model with  $\Omega_m = 0.3$  probe a linear scale of  $0.3 h^{-1} \text{ Mpc}$ . While the assumptions going into the Press-Schechter theory are suspect, numerous numerical simulations have demonstrated that halo number density does seem to be governed by the *linear* power spectrum. A deficit of low mass halos thus implies either additional physics or a deficit of linear theory power on small length scales. This modification could come about either by variations in the primordial power spectrum (e.g., from inflation) or in the cosmological processing of this power spectrum (e.g., from ‘warm’ dark matter).

Recently Kamionkowski & Liddle (1999) pointed out that a well studied class of inflationary models (BSI; see e.g., Starobinsky 1992) could give rise to a deficit of small-scale power in the primordial power spectrum. One way to achieve such a deficit is to introduce a change in the slope of the inflaton potential at a scale determined by the astrophysical problem to be solved, in our case  $k \sim 5 h \text{ Mpc}^{-1}$ . Thus in such models one ‘naturally’ achieves fewer low mass halos than in the conventional inflationary CDM models.

The linear theory power spectrum of these BSI models is well described by Kamionkowski & Liddle (1999). Compared to a scale-invariant model there is a small rise followed by a sharp drop in power at some scale  $k_0$ . Beyond  $k_0$  the power spectrum oscillates with an envelope which falls more steeply than  $k^{-3}$ . At high- $k$  the spectrum recovers to the usual  $k^{-3}$  slope but with much smaller amplitude.

Note that because of this sharp drop, the model near  $k_0$  more closely resembles the familiar top-down scenarios (e.g., HDM) than a bottom-up CDM model for some range of wavenumbers. Thus arguments based on reasoning developed for ‘traditional’ CDM models should be checked against numerical simulations. Further the number density of objects may be one of the only probes of the linear theory power spectrum. Several astrophysical probes of small-scale power are sensitive not to the linear theory but to the non-linear power spectrum. As is well known, objects collapsing under gravitational instability feed power from large scales to small thus allowing small-scale power to be regenerated once a mode goes non-linear. How much power is regenerated requires an actual calculation.

We address several of these issues in the following sections.

## 2. PROBES OF SMALL-SCALE POWER

Any proposal to solve the small halo number density problem by modifying the initial power spectrum must

simultaneously be able to pass other constraints on small-scale power. While we have a number of constraints on the linear and evolved power spectrum on larger scales, there are very few stringent constraints on linear scales below a Mpc. Kamionkowski & Liddle (1999) argue that constraints from the abundance of damped Ly- $\alpha$  systems and the reionization epoch are passed by low-density versions of the BSI model, partly because of the uncertainties involved in making those predictions. The clustering of objects at high- $z$  does not appear to be a promising probe of the matter power spectrum on these small scales. A priori the two most obvious probes are the object abundances which motivated this modification of the power spectrum initially and the power spectrum of the flux in the Ly- $\alpha$  forest.

### 3. SIMULATIONS

To address some of these issues we ran two sets of N-body simulations. The base model in all cases was a  $\Lambda$ CDM model with  $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$ ,  $h = 0.7$ ,  $n = 1$ , COBE normalized using the method of Bunn & White (1997), i.e. with  $\sigma_8 = 0.88$ . The transfer functions were computed using the fits of Eisenstein & Hu (1999) without the baryonic oscillations. This power spectrum was optionally filtered to suppress small-scale power. We have modeled the behavior displayed in Fig. 1 of Kamionkowski & Liddle (1999) with a simple analytic form:

$$\Delta^2(k) \equiv \frac{k^3 P(k)}{2\pi^2} = \left( \Delta_{\text{fid}}^{-2} + (k/k_0)^{3/2} \Delta_{\text{fid}}^{-2}(k_0) \right)^{-1} \quad (1)$$

Here  $\Delta_{\text{fid}}^2$  is the fiducial power spectrum whose high- $k$  behavior we are modifying and the power-law slope of the  $k/k_0$  term was chosen to match the behavior of their Fig. 1 in the range just above  $k_0$ . The model plotted in their Fig. 1 corresponds to  $k_0 \simeq 10h\text{Mpc}^{-1}$ , as shown in our Fig. 2 below.

The first set of simulations used a PM code described in detail in (Meiksin, White & Peacock 1999, White 1999). The simulations used  $256^3$  particles and a  $512^3$  force mesh in a box  $25h^{-1}\text{Mpc}$  on a side evolved from  $z = 70$  to  $z = 3$ . The high mass resolution and quick execution times allowed us to explore parameter space and address the Ly- $\alpha$  forest questions (§4.3) where very high force resolution isn't necessary.

The second set of simulations used a new implementation of a *TreePM* code similar to that described in Bagla (1999). These runs used  $128^3$  particles in the same size box, evolved from  $z = 60$  to  $z = 3$  with the time step dynamically chosen as a small fraction of the local dynamical time. While higher mass resolution would be preferable, this would make the execution time prohibitive on desktop workstations with the current serial version of the code. A spline softened force (Monaghan & Lattanzio 1985, Hernquist & Katz 1989) with  $h = 8 \times 10^{-4} L_{\text{box}} \simeq 20h^{-1}\text{kpc}$  comoving was used (the force was therefore exactly  $1/r^2$  beyond  $h$ ). Very roughly this corresponds to a Plummer law smoothing  $\epsilon \simeq h/3$  (e.g. Springel & White 1999), although a Plummer law gives 1% force accuracy only beyond  $10\epsilon$ .

We have performed numerous tests of the code, among them tests of self-similar evolution of power-law spectra

in critical density models and stable evolution of known halo profiles. The simulations took  $\sim 200$  time steps from  $z = 60$  to  $z = 3$ . Comparison of final particle positions suggested the time step criterion was conservative. We have additionally compared the *TreePM* code with a cosmological Tree code (Springel & White 1999) and found good agreement in the clustering statistics for several different initial conditions, including one of those used here (V. Springel, private communication).

With both the PM and *TreePM* codes we ran 3 realizations of 4 models: the ‘fiducial’  $\Lambda$ CDM power spectrum, and 3 filtered versions with  $k_0 = 10h\text{Mpc}^{-1}$ , closely approximating Fig. 1 of Kamionkowski & Liddle (1999),  $k_0 = 5h\text{Mpc}^{-1}$  and  $k_0 = 2h\text{Mpc}^{-1}$  which show a larger effect more easily resolved by these relatively small simulations. For each of the 3 realizations the same random phases were used for all 4 power spectra to allow inter-comparison. As additional checks on finite volume and resolution effects we also ran simulations in boxes of side  $50h^{-1}\text{Mpc}$  and  $35h^{-1}\text{Mpc}$  finding excellent agreement where the simulations overlapped.

## 4. RESULTS

### 4.1. Visual impression

In Fig. 1 we show slices through the particle distributions of our 4 models. The most extreme model, with  $k_0 = 2h^{-1}\text{Mpc}$  looks markedly different from the others, with smooth low density regions (the simulation initial grid is still clearly visible), and a lack of substructure in the higher density areas. The differences between the other panels are more subtle, and in all cases, are really only apparent on the smallest scales.

### 4.2. Power spectrum

Most probes of small-scale structure, other than the object abundance, depend upon the non-linear power spectrum. The process of gravitational collapse transfers power from large-scales to small, and can generate a  $k^{-3}$  tail in  $P(k)$  if it is absent initially. Fitting formulae for the non-linear power spectrum such as that of Peacock & Dodds (1996) are not applicable for spectra, such as ours, which have regions with  $n < -3$ . We use our PM and *TreePM* simulations to study the non-linear power spectrum.

As can be seen in Fig. 2 the scales of interest are non-linear by  $z = 3$  and small-scale power removed by filtering has been regenerated by collapse of large-scale modes. The fiducial model has good agreement with the fitting function of Peacock & Dodds (1996) on intermediate scales, though for scales smaller than  $k \sim 10h\text{Mpc}^{-1}$  in the *TreePM* simulations we obtain more power than Peacock & Dodds predict by a factor of about 2, independent of the realization or box size. We believe this is due to the very flat nature of the linear theory spectrum on these scales: Jain & Bertschinger (1998) found a similar discrepancy with Peacock & Dodds for  $n = -2$  spectra (see their Fig. 7). We note that other fitting formulae have been developed which it is possible would better reproduce the behaviour of our fiducial model (see e.g. Jain, Mo & White 1995; Ma 1998).

To focus on the dynamics of the power regeneration we show the evolution of the mass power spectrum for

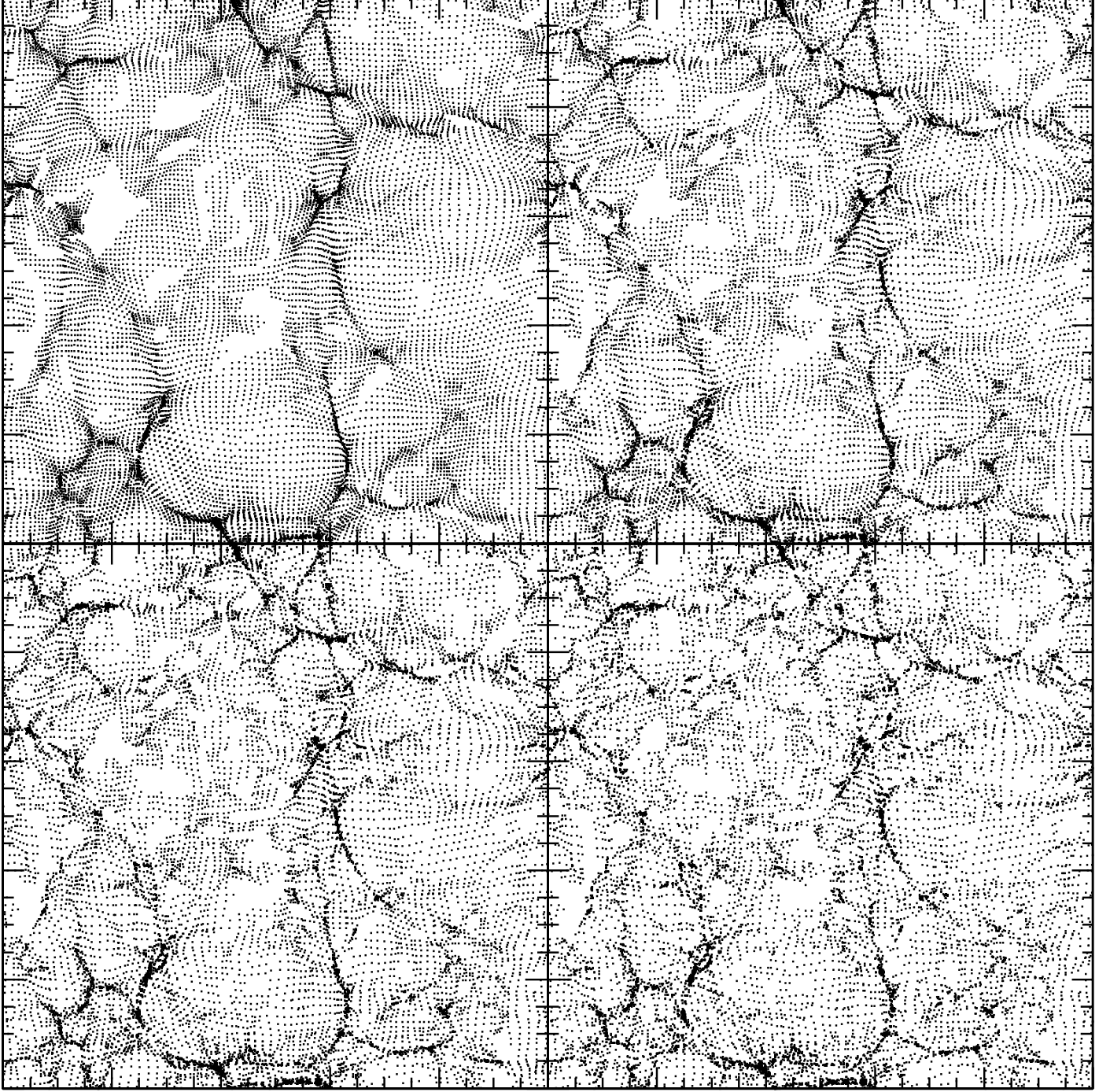


FIG. 1.— Slices through the particle distribution in the *TreePM* simulations of our 4 models at  $z = 3$ . From left to right, the panels show the models with filter scale  $k_0 = 2 \, h \, \text{Mpc}^{-1}$ ,  $k_0 = 5 \, h \, \text{Mpc}^{-1}$  (top row),  $k_0 = 10 \, h \, \text{Mpc}^{-1}$ , and fiducial  $\Lambda\text{CDM}$  (bottom row). The box side-length in each case is  $25 \, h^{-1}\text{Mpc}$  and the slice thickness  $0.25 \, h^{-1}\text{Mpc}$ .

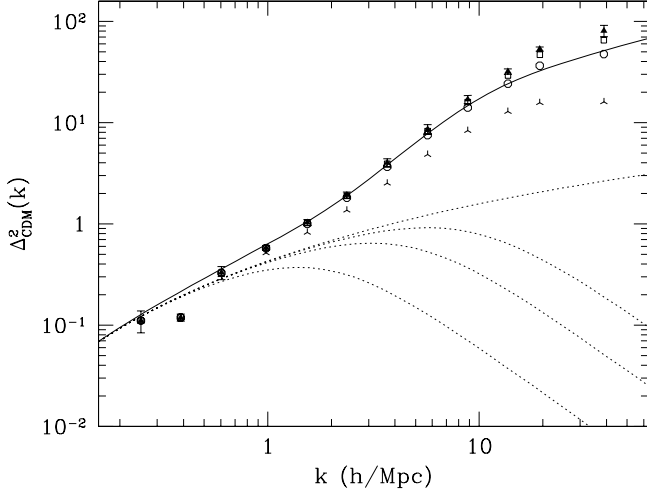


FIG. 2.— The linear and non-linear power spectrum at  $z = 3$ . We show the mean from 3 runs of our *TreePM* simulations for each of 4 models: our fiducial  $\Lambda$ CDM model (triangles), one filtered at  $k_0 = 10 h\text{Mpc}^{-1}$  (open circles),  $k_0 = 5 h\text{Mpc}^{-1}$  (open squares), and  $k_0 = 2 h\text{Mpc}^{-1}$  (three pointed stars). The solid line shows the prediction of Peacock & Dodds (1996) for the fiducial model (the theory is not applicable to the filtered models) and the dotted lines are the linear theory input spectra. While the N-body simulations have the same random phases and so are directly comparable, comparison with the analytic models requires error bars. We show the  $1\sigma$  error on the mean of the fiducial model computed from our 3 realizations. The errors on the other models are similar.

our fiducial model and one filtered model (with  $k_0 = 5 h\text{Mpc}^{-1}$ ) in Fig. 3. We use the average of 3 realizations of PM simulation output here since the greater particle density allows us to probe smaller amplitude fluctuations before shot-noise contamination becomes severe. The PM and *TreePM* simulations agree on the power up to  $k \sim 20 h\text{Mpc}^{-1}$  suggesting we resolve the relevant scales with our PM code.

Notice that even at  $z = 6$ , the “peak” in power introduced by Eq. 1 has disappeared and small-scale power has been regenerated. The difference between the fiducial and filtered models grows progressively smaller as the evolution proceeds. For comparison, the generation of non-linear power has also been studied in numerical experiments by Little, Weinberg & Park (1991), who studied scale-invariant models, and by Bagla & Padmanabhan (1997), amongst others.

Finally it is of interest to ask how the redshift space power spectra evolve. Typically the redshift space spectra appear closer to the linear theory power spectrum than the real space spectra. In Fig. 4 we show the redshift space mass power spectrum as a function of redshift, as in Fig. 3 for the real-space spectra. We can see that even the redshift space spectra have a tail of power at small scales, induced by the non-linear clustering.

#### 4.3. Ly- $\alpha$ forest

There has been a great deal of progress in theoretical understanding of the Ly- $\alpha$  forest recently, due in large part

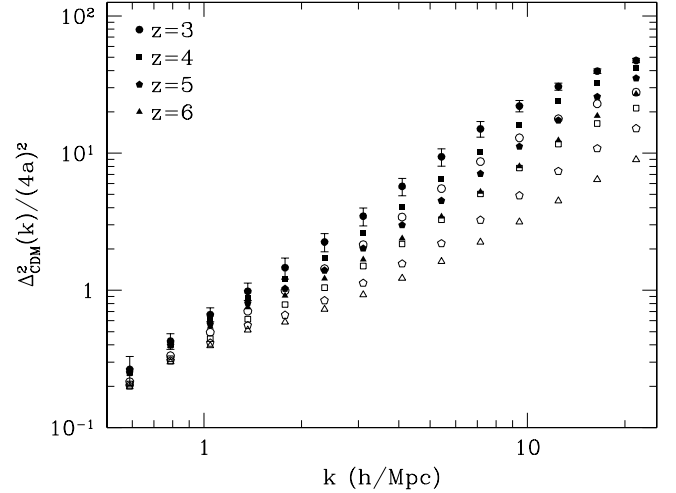


FIG. 3.— The non-linear power spectra as a function of redshift for our fiducial model (solid symbols) and one filtered model with  $k_0 = 5 h\text{Mpc}^{-1}$  (open symbols). The symbols represent the mean of 3 runs of our PM simulations, the error bars show the  $1\sigma$  error on the mean estimated from our 3 realizations at  $z = 0$ . (However recall that for each model the simulations have the same random phases.) The spectra are scaled by  $(4a)^{-2}$  to reduce the effect of linear evolution and highlight the non-linear growth.

to hydrodynamic simulations (Cen et al. 1994; Zhang, Anninos & Norman 1995; Miralda-Escudé et al. 1996; Hernquist et al. 1996; Wadsley & Bond 1996; Zhang et al. 1997; Theuns et al. 1998a, 1998b; Davé et al. 1998; Bryan et al. 1999). In these simulations, it has been found that at high  $z$  ( $> 2$ ), most of the absorption in Ly- $\alpha$  forest spectra is due to a continuous, fluctuating photoionized medium. The physical processes governing this absorbing gas are simple (see e.g., Bi and Davidsen 1997, Hui & Gnedin 1997), and as a result, the optical depth for absorption at a particular point can be related directly to the underlying matter density (Croft et al. 1997). Because of this, observations of the Ly- $\alpha$  forest in quasar spectra can be potentially very useful for probing the clustering of matter.

We have generated simulated Ly- $\alpha$  forest spectra from our PM  $N$ -body outputs at  $z = 3$  in order to test how constraining Ly- $\alpha$  measurements could be for the models described in this paper. To do this, we follow a similar procedure to that outlined in Hui & Gnedin (1998) and Croft et al. (1998). We bin the particle distribution onto  $512^3$  density and velocity grids using a cloud-in-cell scheme, and smooth with a Gaussian filter of width one grid cell. We convert the density in each cell to an optical depth for neutral hydrogen absorption by assuming the ‘Fluctuating Gunn-Peterson Approximation’ (FGPA) (see Croft et al. [1997, 1998]; see also Hui & Gnedin 1998) and assign a temperature to the cell using a power-law density-temperature relation. In all our tests, we use the form  $T = T_0 \rho^{\gamma-1}$ , with  $\gamma = 1.5$  (see Hui & Gnedin 1997 for the expected dependence of  $\gamma$  on reionization epoch). We set the coefficient of proportionality between density and optical depth by requiring that the mean transmitted

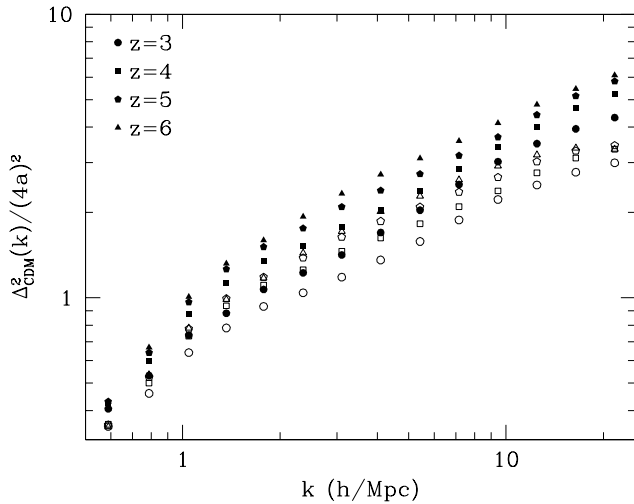


FIG. 4.— The redshift space non-linear power spectra as a function of redshift for our fiducial model (solid symbols) and one filtered model with  $k_0 = 5h \text{ Mpc}^{-1}$  (open symbols). The symbols represent the mean of 3 runs of our PM simulations. The spectra are scaled by  $(4a)^{-2}$  to reduce the effect of linear evolution and highlight the non-linear growth.

flux,  $\langle F \rangle = 0.684$ , in accordance with the observations of McDonald et al. (1999). We run 256 lines of sight parallel to each of the three axes through one simulation box and create mock spectra from a convolution of the optical depths, peculiar velocities, and thermal broadening. The conversion to  $\text{km s}^{-1}$  from  $h^{-1} \text{ Mpc}$  at  $z = 3$  in this model is a factor of 112.

On small scales, the finite pressure of the gas will in detail modify its clustering (see e.g., Hui & Gnedin 1998, Bryan et al. 1999, Theuns et al. 1999), tending to make the gas density field smoother than the dark matter only outputs of our simulations. We have also implemented a 2-species version of ‘Hydro-PM’ (Hui & Gnedin 1998), which takes these effects into account, but find that for our purposes here the main results are adequately reproduced by the pure PM runs. We expect that the details of the spectra that we create will also depend on other assumptions about e.g., the reionization epoch and our simulation methodology. To the extent that we are interested primarily in relative comparisons between models this should not be cause for concern.

For each set of mock spectra we compute the one-dimensional flux power spectrum, using an FFT, and show the results in Fig. 5. In the top panel of this figure, we have

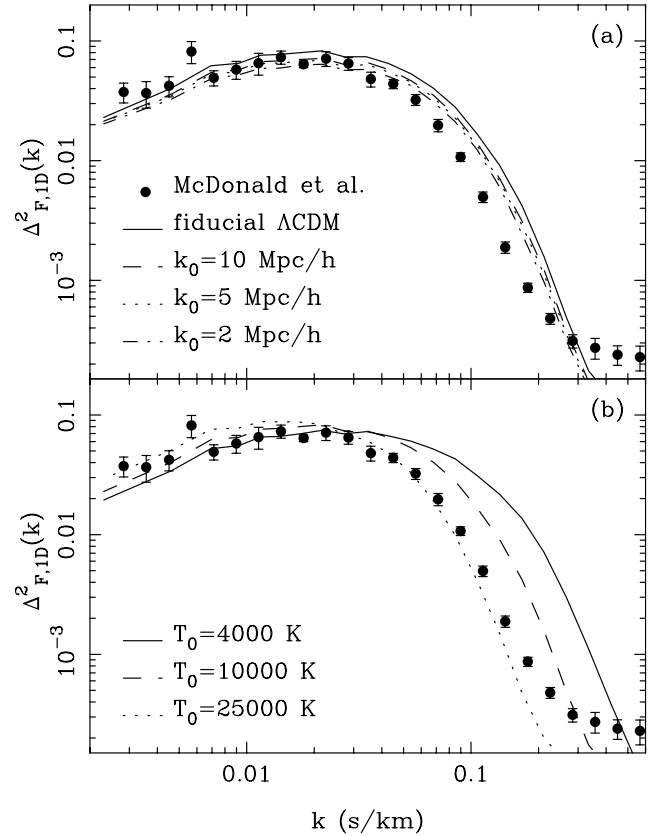


FIG. 5.— (top): The 1D power spectrum of the flux measured from our simulations of the 4 models (lines), all with  $T_0 = 10^4 \text{ K}$ . The observational results, also at  $z = 3$ , of McDonald et al. (1999) are shown as points. (bottom): The 1D power spectrum of the flux for the fiducial model with 3 different values of  $T_0$ .

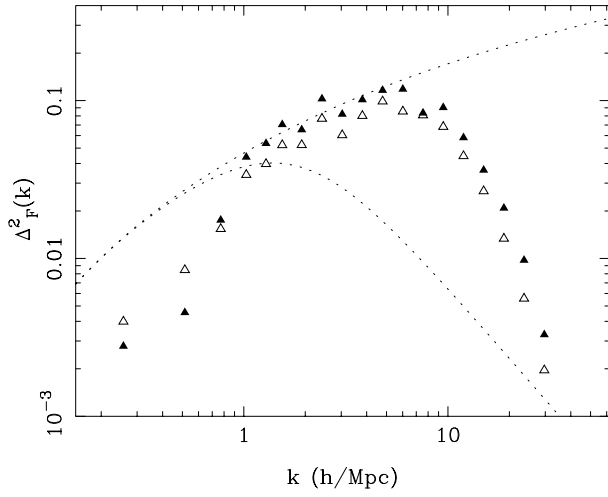


FIG. 6.— The dimensionless 3D power spectrum of the flux measured from our Ly- $\alpha$  PM simulations of 2 of our models: fiducial  $\Lambda$ CDM (filled triangles), and the filtered model with  $k_0 = 2 h \text{Mpc}^{-1}$  (open triangles). The linear theory mass power spectra, scaled down by an arbitrary factor to match the flux power spectra, are shown as dotted lines.

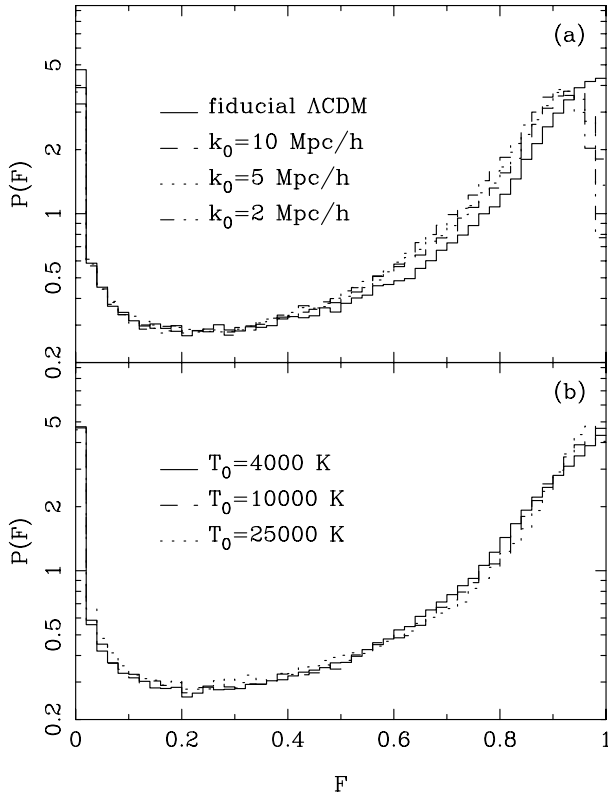


FIG. 7.— Panel (a): The probability distribution function of the flux,  $P(F)$ , measured from our Ly- $\alpha$  PM simulations of the 4 models, all with  $T_0 = 10^4 \text{K}$ . Panel (b):  $P(F)$  for the fiducial  $\Lambda$ CDM model, with three different values for the temperature at the mean density.

set the temperature of the gas at the mean density,  $T_0$ , to be equal to  $10^4 \text{K}$  for all models. The different curves show the effects of linear power suppression at different values of  $k_0$ , while the points are the observational results of McDonald et al. (1999). While a suppression in flux power is seen in Fig. 5, it is very small. If we vary the value of  $T_0$ , the change in the thermal broadening scale causes a more dramatic effect. This can be seen in the lower panel of Fig. 5, where we show results for the fiducial linear power spectrum only. In general, the flux power spectrum shape on small scales will depend on the temperature of the gas through thermal broadening and finite gas pressure, as well as the non-linearity of matter clustering. In the context of this  $\Lambda$ CDM model, it seems as though the observations of McDonald et al. are consistent with a fairly high mean gas temperature, although a more detailed study involving hydrodynamic simulations is needed to give definitive results. What is certain from the present study is that the one-dimensional flux power spectrum provides little constraint on our models with suppressed linear power. Clustering in the flux has apparently been regenerated by non-linear gravitational evolution in a similar fashion to that seen in Fig. 2. There may be a good side to this, though, as insensitivity to the amount of small-scale linear power will mean that estimates of the temperature of the IGM made by looking at small-scale clustering of the flux (as in Fig. 5) should be more robust than expected.

If we assume isotropy of clustering, the three-dimensional flux power spectrum,  $\Delta_F^2(k)$  can be simply recovered from the one-dimensional one (see Croft et al. 1998 for details). It was found by Croft et al. (1998) that on sufficiently large scales the shape of  $\Delta_F^2$  measured from simulated spectra matches well that of the linear theory mass power spectrum,  $\Delta^2(k)$ . In Fig. 6 we test this using the model with  $k_0 = 2 h^{-1} \text{Mpc}$ , and the fiducial model (both with  $T_0 = 10^4 \text{K}$ ). We can see that there is not much difference between  $\Delta_F^2(k)$  for the two (the same is true of the two intermediate models, which we do not plot). The linear theory mass power spectrum (arbitrarily normalized) is shown for comparison. On scales approaching the box size, cosmic variance is large enough to account for the difference between the linear curve and the points. On smaller scales, there is still scatter, but the points taken from the simulation with less linear power are systematically a bit lower. We might expect the simulation points to start to trace the linear theory shape around the scale of non-linearity, where  $\Delta^2(k)$  becomes comparable to 1, which from Fig. 2 is around  $k \sim 1 - 2 h \text{Mpc}^{-1}$ . If we look at Fig. 6, this does seem reasonable, and we do find similar results even if we assume different gas temperatures. On smaller scales though,  $\Delta_F^2(k)$  has been regenerated by non-linearity, so that the exact relationship between  $\Delta_F^2(k)$  and the matter clustering is complex, and as in Fig. 5 the differences between models small.

Another statistic which we can check, in order to see whether suppression of linear power has caused any changes in higher-order clustering, is the probability distribution of the flux. We plot this in Fig. 7, showing the 4 models with different linear power (and all with  $T_0 = 10^4 \text{K}$ ) in the top panel. There are small differences between the models, particularly at the high flux end, where the models with more power appear to have more truly empty

regions. These small differences are likely to remain unobserved though due to the difficulty of accurate continuum fitting. The lower panel of this figure shows results for our fiducial model with different gas temperatures. There appears to be little difference between the curves, although we have found that differences do appear if the spectra are subjected to a moderate amount of smoothing (e.g. with a  $50 \text{ km s}^{-1}$  Gaussian; not shown).

From our tests with both sets of statistics, we find that the Ly- $\alpha$  forest is not a promising discriminator between the models we are considering here. Two effects conspire to mask any differences in the Ly- $\alpha$  measurements on the small scales where there are large differences in the linear power spectra. First, the thermal broadening has the effect of smoothing the spectra (the thermal width of features is a few 10s of  $\text{km s}^{-1}$ ). Second, non-linear evolution of the density field causes power to be rapidly transferred from large to small scales. For these models the scale of non-linearity at  $z = 3$  is about the same or larger than the scale at which there are large differences in the linear power spectra. On smaller scales, the shape of the three-dimensional flux power spectrum no longer follows that of the linear mass power spectrum.

#### 4.4. Halo abundance

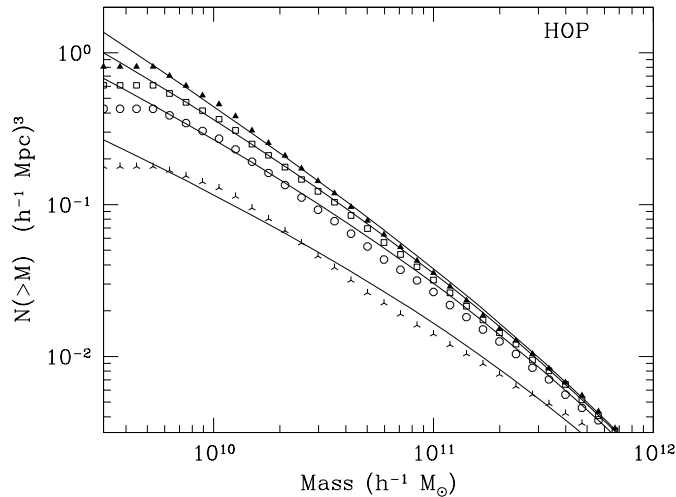


FIG. 8.— The mass functions of our simulations, using the HOP algorithm of Eisenstein & Hut (1998) to find halos. We show results for fiducial model (triangles) and our 3 models with reduced small-scale power:  $k_0 = 10 h\text{Mpc}^{-1}$  (open squares),  $k_0 = 5 h\text{Mpc}^{-1}$  (open circles), and  $k_0 = 2 h\text{Mpc}^{-1}$  (three pointed stars). We have co-added the mass functions of the three realizations to construct an “average” mass function – the error on the mean as calculated from the 3 realizations is smaller than the size of the symbols. The solid lines show the predictions of the Press-Schechter theory. We have used top-hat smoothing and  $\delta_c = 2$  in the calculation of the Press-Schechter predictions.

Ideally we would like to evolve a large volume and study the number density of small halos present today within a larger halo such as the Milky Way. This is not possible with the limited dynamic range of the simulations presented here. While many effects *could* potentially disturb all the small halos as they interact with each other and a larger halo, very high resolution numerical simulations suggest that this may not be the case in practice. Moore et al. (1999a) find that a large number of small halos are not disrupted, so that the number remaining will still be a substantial fraction of the number that existed in the proto-galaxy. In this paper, we focus on the number density of halos in our simulations at  $z = 3$  where our  $25 h^{-1}\text{Mpc}$  box is just about to go non-linear. We assume that these small halos would become incorporated into a larger halo at later times by the usual evolution of clustering, and that the fraction that survive disruption can be predicted by the referring to the detailed calculations of Moore et al. (1999a). Here we are only interested in deficit in the number of small halos in our suppressed models relative to the fiducial model. This relative fraction should be similar at the redshift of our simulation box to what it would be at  $z = 0$ , although the absolute number of halos could only be quantified using simulations like those of Moore et al. Similar assumptions to ours were also used by Kamionkowski & Liddle (1999).

We show in Fig. 8 the Press-Schechter predictions for our fiducial and filtered models and the numerically determined mass functions. As there is no perfect algorithmic definition of a “group” of points, the mass function is sensitive to a small degree to the halo finding algorithm. We have used both the Friends-of-Friends (FOF; Davis et al. 1985) algorithm, with linking length 0.2, and the HOP (Eisenstein & Hut 1998) halo finding algorithm to construct these mass functions. We find that the mass function differed slightly if we changed the parameters in the algorithms or the algorithm used, and show results for

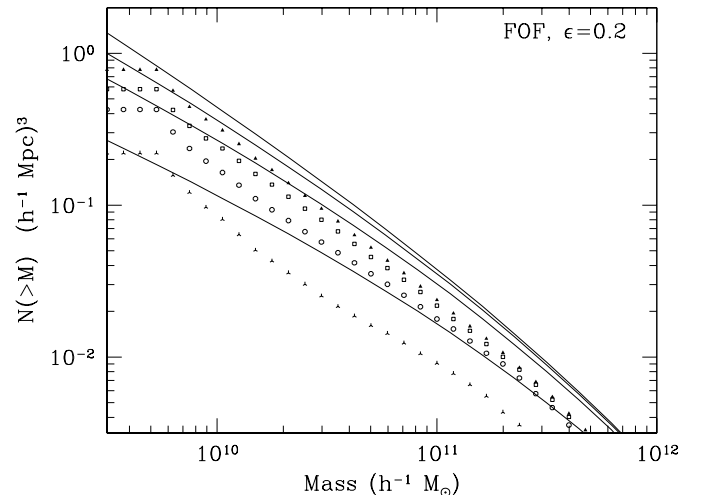


FIG. 9.— The simulation mass functions, as for Fig. 8, except that a friends-of-friends scheme (with linking length  $\epsilon = 0.2$ ) was used to find halos.

both of these schemes. However, these differences should not affect our main conclusions, as we are interested in comparing models with different amounts of small-scale power to each other.

The N-body mass functions and the Press-Schechter predictions are shown in Figs. 8 and 9 for the HOP and FOF algorithms respectively. The “shelf” at the low mass end of the N-body mass functions arises because of the minimum number of particles allowed to form a group. There are no very low mass halos in the simulation. If we use  $\delta_c = 1.69$  with a top-hat window in the Press-Schechter predictions we find that the mass functions are considerably shallower, in the sense of having too many large-mass halos, than the HOP N-body results for both  $z = 4$  and 3. The two can be brought into better agreement if we increase  $\delta_c$  to 2 (c.f. Gelb & Bertschinger 1994) as we have done. Even with this modification, the Press-Schechter predictions still overestimate the FOF results by a factor of up to 2. To check for simulation artifacts we also ran several larger boxes. We find that the mass functions from a sequence of larger boxes (up to  $50h^{-1}\text{Mpc}$ ) with different random phases match smoothly and stably onto the mass function of these simulations, suggesting that there are no finite volume or sample variance effects operating. As a final check, a completely separate analysis chain using a different N-body code (a cosmological Tree code) obtains the same mass function at  $z = 3$  for one of our runs (V. Springel; private communication).

Figs. 8 and 9 suggest that the number of halos is indeed governed by the linear theory power spectrum. The amount of suppression relative to the fiducial model is robust to the parameters of our group finding algorithm or the algorithm used. The absolute number of halos can in principle be predicted from statistics of the initial density field, although there are uncertainties related to the definition of halos in the simulations and parameters in Press-Schechter theory.

We find that in order to reduce the number of small halos by a large factor (for example Kamionkowski & Liddle (1999) recommend about an order of magnitude), we require a fairly severe filtering of the fiducial model, using a filter with  $k_0 = 2 h \text{Mpc}^{-1}$ .

Finally we remark that this set of simulations does not have enough mass resolution to probe the structure of the halos we find. However simulations by Moore et al. (1999b) suggest that the halo structure will not be sensitive to the filtering of the initial power spectrum. This lends some support to our assumption that the amount of disruption of the small halos when they become incorporated into a larger halo does not depend on the alterations we have made to the initial power spectrum.

## 5. CONCLUSIONS

While the essential picture of hierarchical formation of large-scale structure in a universe containing primarily cold dark matter appears to work well, some puzzles remain. One of these is the paucity of dwarf galaxies in the local neighborhood. One resolution of this “lack-of-small-halos problem” is a modification of the initial power spectrum, reducing the amount of small-scale power. There exist inflationary models which can accomplish this, though the scale of the modification must be put in by hand. Other approaches, such as assuming that the universe is

dominated by Warm Dark Matter (WDM), will have a similar effect (and both approaches may solve other problems: see e.g. Sommer-Larsen & Dolgov 1999). In models with reduced small-scale power structure forms in a top-down manner over a range of scales near the break, so ansatz developed for the “traditional” bottom-up scenario should be treated with caution. In this work we have used numerical simulations to address the question of how one could constrain such a modification of the initial power spectrum. We note that we have dealt in detail only with a model with suppressed initial power. In a WDM model the deficit of power arises from the dark matter velocity dispersion, and so such a model may behave slightly differently, at least on the smallest scales.

We find that the halo mass function depends primarily on the linear theory power spectrum, so a suppression of small-scale power does reduce the number of low mass halos. While the Press-Schechter theory predicts qualitatively the right behavior, its free parameter ( $\delta_c$ ) must be adjusted to fit the N-body results. To reduce the number of  $10^{10} M_\odot$  halos by a factor of  $> 5$  compared to our fiducial model requires a fairly extreme filtering of the primordial power spectrum, and the structure that forms in such a model appears qualitatively different to the fiducial  $\Lambda\text{CDM}$  model (Fig. 1).

Collapse of large-scale structures as they go non-linear regenerates a “tail” in  $P(k)$  if it is suppressed in the initial conditions (and this holds in redshift as well as real space). Thus probes which measure primarily the evolved power spectrum are less sensitive to reduced small-scale power than one might think. We particularly examine measurements of clustering from the Ly- $\alpha$  forest flux. On the scales which govern the number of small halos, choosing a different gas temperature affects Ly- $\alpha$  clustering much more strongly than suppressing the linear power spectrum. The matter power spectrum measurement made from the low resolution Ly- $\alpha$  forest spectra by Croft et al. (1998) probes scales just above this, which are still linear, and offers essentially little constraint on these models. Any extension of these simple Ly- $\alpha$  forest measurements to smaller scales must necessarily have less general conclusions drawn from them.

Given that the number density of collapsed objects seems to be the most sensitive probe of this small-scale modification of the power spectrum, other observations which depend on this should be used to make consistency checks. At the moment, the obvious choices, such as the number density of damped Ly- $\alpha$  systems, or the redshift of reionization induced by the formation of the first stars and quasars, are difficult to predict accurately from theory. Their potentially strong discriminatory power will make them useful eventually though, as we learn whether more of Cosmology’s puzzles can be resolved by an absence of small-scale power.

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